Optimization

Homework 3

(Due Day: 9:00 AM, Dec 17, 2008, hardcopies in the class)

1. Convert the following linear programming problem to *standard form*:

$$\begin{array}{ll} \text{maximize} & 2x_1 + x_2 \\ \text{subject to} & 0 \le x_1 \le 2 \\ & x_1 + x_2 \le 3 \\ & x_1 + 2x_2 \le 5 \\ & x_2 \ge 0. \end{array}$$

2. Consider the system of equations:

$$\begin{bmatrix} 2 & -1 & 2 & -1 & 3 \\ 1 & 2 & 3 & 1 & 0 \\ 1 & 0 & -2 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 14 \\ 5 \\ -10 \end{bmatrix}.$$

Check if the system has basic solutions. If yes, find all basic solutions.

3. Consider the following standard form LP problem:

$$\begin{array}{ll} \text{minimize} & 2x_1 - x_2 - x_3 \\ \text{subject to} & 3x_1 + x_2 + x_4 = 4 \\ & 6x_1 + 2x_2 + x_3 + x_4 = 5 \\ & x_1, x_2, x_3, x_4 \ge 0. \end{array}$$

a. Write down the $\boldsymbol{A}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ matrices/vectors for the problem.

b. Consider the basis consisting of the third and fourth columns of $\boldsymbol{A}$, ordered according to $[\boldsymbol{a}_4, \boldsymbol{a}_3]$. Compute the canonical tableau corresponding to this basis.

c. Write down the basic feasible solution corresponding to the above basis, and its objective function value.

d. Write down the values of the reduced cost coefficients (for all the variables) corresponding to the above basis.

e. Is the basic feasible solution in part c an optimal feasible solution? If yes, explain why. If not, determine which element of the canonical tableau to pivot about so that the new basic feasible solution will have a lower objective function value.

f. Suppose we apply the two-phase method to the problem, and at the end of phase I, the tableau for the artificial problem is

$$\begin{array}{ccccccc} 0 & 0 & -1 & 1 & 2 & -1 & 3 \\ 1 & \frac{1}{3} & \frac{1}{3} & 0 & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{array}$$

Does the original problem have a basic feasible solution? Explain.

g. From the final tableau for phase I in part f, find the initial canonical tableau for phase II.

4. Use the simplex method to solve the following linear program:

$$\begin{array}{ll} \text{maximize} & x_1 + x_2 + 3x_3 \\ \text{subject to} & x_1 + x_3 = 1 \\ & x_2 + x_3 = 2 \\ & x_1, x_2, x_3 \geq 0. \end{array}$$

5. Consider the linear program:

$$\begin{array}{ll} \text{maximize} & 2x_1 + x_2 \\ \text{subject to} & 0 \leq x_1 \leq 5 \\ & 0 \leq x_2 \leq 7 \\ & x_1 + x_2 \leq 9. \end{array}$$

Convert the problem to standard form and solve it using the simplex method.

6. **In the revised simplex method, explain why the situation is unbounded when no $y_{iq} > 0$.**

7. Consider the linear program

$$\begin{array}{ll} \text{minimize} & 4x_1 + 3x_2 \\ \text{subject to} & 5x_1 + x_2 \geq 11 \\ & 2x_1 + x_2 \geq 8 \\ & x_1 + 2x_2 \geq 7 \\ & x_1, x_2 \geq 0. \end{array}$$

Write down the corresponding dual problem, and find the solution to the dual. (Compare the above problem with the one in Exercise 16.8, part a.)

8. Consider the linear programming problem

$$\begin{array}{ll} \text{maximize} & \boldsymbol{c}^T \boldsymbol{x} \\ \text{subject to} & \boldsymbol{A}\boldsymbol{x} \leq \boldsymbol{0} \\ & \boldsymbol{x} \geq \boldsymbol{0}, \end{array}$$

where $\boldsymbol{c} = [1, 1, \ldots, 1]^T$. Assume that the problem has a solution.

a. Write down the dual of the above problem.

b. Find the solution to the above problem.

c. What can you say about the constraint set for the above problem?